Application-Oriented Finite Sample Experiment Design: A Semidefinite Relaxation Approach

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Abstract: In this paper, the problem of input signal design with the property that the estimated model satisfies a given performance level with a prescribed probability is studied. The aforementioned performance level is associated with a particular application. This problem is well-known to fall within the class of chance-constrained optimization problems, which are nonconvex in most cases. Convexification is attempted based on a Markov inequality, leading to semidefinite programming (SDP) relaxation formulations. As applications, we focus on the identification of multiple input multiple output (MIMO) wireless communication channel models for minimum mean square error (MMSE) channel equalization and zero-forcing (ZF) precoding.

1. INTRODUCTION

A basic subproblem in the context of system identification is that of experiment design. Overviews of this topic over the last decade can be found in [Gevers, 2005, Hjalmarsson, 2005, Pronzato, 2008, Hjalmarsson, 2009]. Contributions include convexification [Jansson and Hjalmarsson, 2005], robust design [Rojas et al., 2007], least-costly design [Bombois et al., 2006], and closed versus open loop experiments [Agüero and Goodwin, 2007].

An intuitive consideration of the experiment design problem is that of application-oriented input design. In this context, we depart from the usual philosophy of designing the experiment subject to a measure, which quantifies the distance of the estimated model from the real one. Here, the experiment is designed to optimize a performance metric associated with the particular application where the estimated model will be used, [Gevers and Ljung, 1986, Forssell and Ljung, 2000, Barenthin et al., 2008]. A conceptual framework for application-oriented experiment design was outlined in [Hjalmarsson, 2009]. The framework hinges on introducing a function J, which quantifies the degradation in performance, when a model that differs from the true system is used in the design of the application. Suppose that the performance is deemed acceptable if $J \leq 1/\gamma$ for some parameter γ, which we will call *accuracy*. Clearly, J is dependent on a model G and the set of admissible models is denoted as $\mathcal{E}_{adm} = \{G : J \leq 1/\gamma\}$. The system identification objective is then to select a model in $\mathcal{E}_{adm}$. Therefore, the least-costly experiment is given as

$$\begin{aligned} \min_{\text{Experiment}} \quad & \text{Experimental effort} \\ \text{s.t.} \quad & \hat{G} \in \mathcal{E}_{adm}, \end{aligned} \tag{1}$$

where $\hat{G}$ is the identified model. For the experimental effort, different measures commonly used are input or output power, and experimental length.

* This work was partially supported by the European Research Council under the advanced grant LEARN, contract 267381 and by the Swedish Research Council under contract 621-2010-5819.

In this paper, we assume parametric model identification, where the parameters are allowed to take complex values. Thus, our model is represented by a parameter vector $\mathbf{h} \in \mathbb{C}^n$ and its estimate by $\widehat{\mathbf{h}} \in \mathbb{C}^n$. The unknown model can be estimated either using a deterministic or a Bayesian estimator. In the first case, we use the minimum variance unbiased (MVU) estimator and in the second case the MMSE estimator, [Kay, 1993]. For the MVU case, the parameter estimator has the distribution:

$$\widehat{\mathbf{h}} \sim \mathcal{CN}(\mathbf{h}, \mathcal{I}_{\mathrm{F,MVU}}^{-1}), \tag{2}$$

while for the MMSE case, the posterior parameter distribution is:

$$\mathbf{h}\,|\mathbf{y}, \mathbf{u} \sim \mathcal{CN}(\widehat{\mathbf{h}}, \mathcal{I}_{\mathrm{F,MMSE}}^{-1}). \tag{3}$$

In the last two equations, $\mathcal{CN}(\bar{\mathbf{x}}, \mathbf{Q})$ stands for the circularly symmetric complex Gaussian distribution with mean $\bar{\mathbf{x}}$ and covariance $\mathbf{Q}$, $\mathcal{I}_{\mathrm{F,MVU}}$ [Kay, 1993] and $\mathcal{I}_{\mathrm{F,MMSE}}$ are the inverse covariance matrices for the MVU and MMSE estimators, respectively and $\mathbf{u} \in \mathbb{C}^{n_T \times B}$, $\mathbf{y} \in \mathbb{C}^{n_R \times B}$ are the (complex-valued) input and output data. More details will be provided in Section 3.

Under (2) and (3), (1) can only be guaranteed with a prescribed probability. We therefore relax (1) to

$$\begin{aligned} \min_{\text{Experiment}} \quad & \text{Experimental effort} \\ \text{s.t.} \quad & \mathbf{P}_{\mathcal{X}}\{\widehat{\mathbf{h}} \in \mathcal{E}_{adm}\} \geq 1 - \varepsilon \end{aligned} \tag{4}$$

where $\mathbf{P}_{\mathcal{X}}\{Y\}$ is the probability of the event Y over the probability space corresponding to $\mathcal{X}$. For the MVU estimator, $\mathcal{X}$ corresponds to $\widehat{\mathbf{h}}$, while for the MMSE estimator $\mathcal{X}$ corresponds to $(\widehat{\mathbf{h}}, \mathbf{h})$.

From the definition of $\mathcal{E}_{adm}$ and with the rest of our assumptions, it is clear that the chance constraint in the last two problems can be written as $\mathbf{P}_{\mathcal{X}}\{J(\widetilde{\mathbf{h}}, \mathbf{h}) \geq 1/\gamma\} \leq \varepsilon$, where $\widetilde{\mathbf{h}} = \widehat{\mathbf{h}} - \mathbf{h}$ denotes the parameter estimation error. This constraint is typically nonconvex. Therefore, further relaxations are required. In [Hjalmarsson, 2009], the chance constraint is replaced by a convex linear matrix inequality (LMI) equivalent to the relation $\mathcal{E}_{id} \subseteq \mathcal{E}_{adm}$, where $\mathcal{E}_{id}$ corresponds to a confidence ellipsoid for $\hat{G}$. Alternative convex relaxations have been subsequently suggested in [Rojas et al., 2011]. In this paper, following [Rojas et al., 2011], the Markov inequality is employed to approximate the chance constraint in (4), while maintaining at the same time attractive computational properties. The resulting approximations will be further manipulated and relaxed to yield semidefinite programming (SDP) relaxations of (4).

It is worth noting that the use of SDP relaxations has already been proposed in [Manchester, 2010] to solve input design problems in the time domain subject to amplitude constraints. However, the SDP relaxations employed in this paper differ from those in [Manchester, 2010], due to the Kronecker structure of the MIMO communications problem considered here.

The paper is organized as follows. In Section 2, following [Rojas et al., 2011] the Markov inequality is used to relax the chance constraint of (4). Section 3 introduces the problem of input design for MIMO communication systems. In Section 4 we study how to convexify the Markov bound relaxation derived in Section 2 for the Kronecker structures present in MIMO communication systems. Section 5 shows how this relaxation can be used to perform input design for MMSE channel equalization and ZF precoding. Some simulation examples are given in Section 6. Finally, the paper is concluded in Section 7.

Notation: ^{T}, ^{H} and * denote the transposition, Hermitian transposition and complex conjugation operators. $\|\cdot\|$ is the Euclidean norm of a (complex) vector or a (complex) matrix. For a matrix $\mathbf{A}$, $\mathbf{A}_{i,j}$ denotes its (i,j)th element. Finally, $\text{vec}(\cdot)$ denotes the vectorization of a matrix, i.e., the stacking of its columns into a single vector.

2. MARKOV BOUND APPROXIMATION

A possible convex approximation of the chance constraint can be based on the Markov inequality [Papoulis, 1991]. To this end, we assume that J is or can be approximated by a quadratic form with respect to $\widetilde{\mathbf{h}}$:

$$J(\widetilde{\mathbf{h}},\mathbf{h}) = \widetilde{\mathbf{h}}^H \boldsymbol{\mathcal{I}}_{adm}(\mathbf{h})\widetilde{\mathbf{h}},$$

where $\boldsymbol{\mathcal{I}}_{adm}(\mathbf{h})$ is a Hermitian positive semidefinite matrix possibly dependent on $\mathbf{h}$. To continue with the convex approximation, it is necessary to approximate $\mathbf{h}$ in $\boldsymbol{\mathcal{I}}_{adm}(\mathbf{h})$ by a previous estimate of $\mathbf{h}$, say $\widehat{\mathbf{h}}_o$, turning in this way $\boldsymbol{\mathcal{I}}_{adm} = \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)$ into a deterministic matrix both in the case of the MVU and the MMSE estimators. Clearly, this implies that the evolution of the true parameter vector $\mathbf{h}$ with respect to the observation intervals of the system is such that an approximation of this form approximately holds.

Using the Markov inequality, the chance constraint can be approximated as follows:

$$\begin{aligned}\mathbf{P}_{\mathcal{X}}\{J(\widetilde{\mathbf{h}},\mathbf{h}) \geq 1/\gamma\} &\leq \gamma \mathbf{E}_{\mathcal{X}}\{\widetilde{\mathbf{h}}^H \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)\widetilde{\mathbf{h}}\} \\ &= \gamma \text{Tr}[\boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)\mathbf{E}_{\mathcal{X}}\{\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}^H\}] \\ &= \gamma \text{Tr}[\boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)\boldsymbol{\mathcal{I}}_{\text{F}}^{-1}],\end{aligned}$$

where $\boldsymbol{\mathcal{I}}_{\text{F}}$ is ether $\boldsymbol{\mathcal{I}}_{\text{F,MVU}}$ or $\boldsymbol{\mathcal{I}}_{\text{F,MMSE}}$ depending on the employed estimator. The chance constraint will be satisfied if

$$\gamma \text{Tr}[\boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)\boldsymbol{\mathcal{I}}_{\text{F}}^{-1}] \leq \varepsilon.$$

This condition can also be written as an LMI in $\boldsymbol{\mathcal{I}}_{\text{F}}$, by using the Schur complement [Boyd et al., 1994]:

$$\text{Tr}[\mathbf{M}] \leq \frac{\varepsilon}{\gamma}, \quad \begin{bmatrix} \mathbf{M} & \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{1/2} \\ \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{H/2} & \boldsymbol{\mathcal{I}}_{\text{F}} \end{bmatrix} \geq 0. \quad (5)$$

Here, $\mathbf{M} = \mathbf{M}^H \in \mathbb{C}^{n\times n}$ is an auxiliary (free) matrix, and $\boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o) = \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{H/2}\boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{1/2}$.

In the following, we will focus on the specific applications of interest, namely that of MMSE channel equalization and ZF precoding in MIMO wireless communication systems, and we will show how $\boldsymbol{\mathcal{I}}_{\text{F}}$ is related to the desired input signal and how (5) can be used to provide semidefinite approximations of (4).

3. MIMO SYSTEM MODEL

We consider a MIMO communication system with n_T antennas at the transmitter and n_R antennas at the receiver [Paulraj et al., 2003]. The received signal at time t is modelled as

$$\mathbf{y}(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{n}(t)$$

where $\mathbf{x}(t) \in \mathbb{C}^{n_T}$ and $\mathbf{y}(t) \in \mathbb{C}^{n_R}$ are the baseband representations of the transmitted and received signals, respectively. The impact of background noise and interference from adjacent communication links is represented by the additive term $\mathbf{n}(t) \in \mathbb{C}^{n_R}$. We will further assume that $\mathbf{x}(t)$ and $\mathbf{n}(t)$ are both (weakly) stationary signals. The channel response is modeled by $\mathbf{H} \in \mathbb{C}^{n_R\times n_T}$, which is assumed constant during the transmission of one block of data. In the context of either the MVU or the MMSE estimators, two different models of the channel will be considered:

i) A deterministic model.
ii) A stochastic Rayleigh fading model, i.e. $\text{vec}(\mathbf{H}) \in \mathcal{CN}(\mathbf{0},\mathbf{R})$, where, for mathematical tractability, we will assume that the known covariance matrix $\mathbf{R}$ possesses the Kronecker model used, e.g., in [Liu et al., 2007, Biguesh et al., 2009]:

$$\mathbf{R} = \mathbf{R}_T^T \otimes \mathbf{R}_R$$

where $\mathbf{R}_T \in \mathbb{C}^{n_T\times n_T}$ and $\mathbf{R}_R \in \mathbb{C}^{n_R\times n_R}$ are the spatial covariance matrices at the transmitter and receiver side, respectively. Here, $\otimes$ denotes the Kronecker product [Brewer, 1978] . This model has been experimentally verified in [Kermoal et al., 2002, Yu et al., 2004] and in [Gazor and Rad, 2006, Rad and Gazor, 2008], where it is argued that this Kronecker structure is reasonable, since the antenna size is significantly smaller than the distance between the transmitter and the receiver.

We consider training signals of arbitrary length B, represented by $\mathbf{P} \in \mathbb{C}^{n_T\times B}$, whose columns are the transmitted signal vectors during training. Placing the received vectors in $\mathbf{Y} = [\mathbf{y}(1) \ldots \mathbf{y}(B)] \in \mathbb{C}^{n_R\times B}$, we have:

$$\mathbf{Y} = \mathbf{HP} + \mathbf{N},$$

where $\mathbf{N} = [\mathbf{n}(1) \ldots \mathbf{n}(B)] \in \mathbb{C}^{n_R\times B}$ is the combined noise and interference matrix.

Defining $\widetilde{\mathbf{P}} = \mathbf{P}^T \otimes \mathbf{I}$, we can then write

$$\text{vec}(\mathbf{Y}) = \widetilde{\mathbf{P}}\,\text{vec}(\mathbf{H}) + \text{vec}(\mathbf{N}). \quad (6)$$

As, for example, in [Biguesh et al., 2009, Liu et al., 2007], we assume that $\text{vec}(\mathbf{N}) \in \mathcal{CN}(\mathbf{0},\mathbf{S})$, where the covariance matrix $\mathbf{S}$ also possesses a Kronecker structure:

$$\mathbf{S} = \mathbf{S}_Q^T \otimes \mathbf{S}_R.$$

Here, $\mathbf{S}_Q \in \mathbb{C}^{B\times B}$ represents the temporal covariance matrix [1] and $\mathbf{S}_R \in \mathbb{C}^{n_R\times n_R}$ represents the received spatial covariance matrix.

In the case of the MMSE estimator, the channel and noise statistics will be assumed known to the receiver during estimation, while in the case of the MVU estimator only the noise statistics will be considered known. Statistics can often be achieved by long-term estimation and tracking [Werner and Jansson, 2009].

For the data transmission phase, we will assume that the transmit signal $\{\mathbf{x}(t)\}$ is a zero-mean, weakly stationary process, which is both temporally and spatially white, i.e., its spectrum is $\mathbf{\Phi}_x(\omega) = \lambda_x \mathbf{I}$.

As far as the different ways to estimate the MIMO channel $\mathbf{H}$ are concerned, the MVU channel estimator for the signal model (6), subject to a deterministic channel (Assumption i), is given by:

$$\mathrm{vec}(\widehat{\mathbf{H}}_{\mathrm{MVU}}) = (\widetilde{\mathbf{P}}^H \mathbf{S}^{-1} \widetilde{\mathbf{P}})^{-1} \widetilde{\mathbf{P}}^H \mathbf{S}^{-1}\,\mathrm{vec}(\mathbf{Y}).$$

For this estimate, the inverse covariance is

$$\boldsymbol{\mathcal{I}}_{\mathrm{F,MVU}} = \widetilde{\mathbf{P}}^H \mathbf{S}^{-1} \widetilde{\mathbf{P}}.$$

For the case of a stochastic channel model (Assumption ii), the first and second moments of the posterior parameter vector are

$$\mathrm{vec}(\widehat{\mathbf{H}}_{\mathrm{MMSE}}) = (\mathbf{R}^{-1} + \widetilde{\mathbf{P}}^H \mathbf{S}^{-1} \widetilde{\mathbf{P}})^{-1} \widetilde{\mathbf{P}}^H \mathbf{S}^{-1}\,\mathrm{vec}(\mathbf{Y})$$
$$\boldsymbol{\mathcal{I}}^{-1}_{\mathrm{F,MMSE}} = (\mathbf{R}^{-1} + \widetilde{\mathbf{P}}^H \mathbf{S}^{-1} \widetilde{\mathbf{P}})^{-1}.$$

4. SDP RELAXATIONS BASED ON THE MARKOV BOUND

The experimental effort of interest in our context is assumed to be the input power. Focusing on the case of the MVU estimator and using (5), the optimization problem (4) is relaxed as follows:

$$\begin{aligned}\min_{\mathbf{P},\mathbf{M}}\ & \mathrm{Tr}\left[\mathbf{P}\mathbf{P}^H\right] = \mathrm{Tr}\left[\mathbf{P}^*\mathbf{P}^T\right] \\ \text{s.t.}\ & \mathrm{Tr}\left[\mathbf{M}\right] \le \frac{\varepsilon}{\gamma},\ \begin{bmatrix} \mathbf{M} & \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{1/2} \\ \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{H/2} & \mathbf{P}^*\mathbf{S}_Q^{-T}\mathbf{P}^T \otimes \mathbf{S}_R^{-1}\end{bmatrix} \ge 0.\end{aligned}$$

Here, $\mathbf{M} = \mathbf{M}^H \in \mathbb{C}^{n_T n_R \times n_T n_R}$ is an auxiliary (free) matrix and $\widehat{\mathbf{h}}_o = \mathrm{vec}(\widehat{\mathbf{H}}_{\mathrm{MVU}})_o$, i.e., a previous MVU estimate of the unknown MIMO channel.

We set $\mathbf{P}_Q = \mathbf{P}^*\mathbf{S}_Q^{-T/2}$. Using an additional free variable $\beta \in \mathbb{R}$, the last optimization problem takes the form

$$\begin{aligned}\min_{\beta,\mathbf{P}_Q,\mathbf{M}}\ & \beta \\ \text{s.t.}\ & \mathrm{Tr}[\mathbf{M}] \le \frac{\varepsilon}{\gamma},\quad \mathrm{Tr}\left[\mathbf{P}_Q\mathbf{S}_Q^T\mathbf{P}_Q^H\right] \le \beta \\ & \begin{bmatrix} \mathbf{M} & \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{1/2} \\ \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{H/2} & \mathbf{P}_Q\mathbf{P}_Q^H \otimes \mathbf{S}_R^{-1}\end{bmatrix} \ge 0.\end{aligned} \tag{7}$$

To achieve a valid semidefinite relaxation of the chance constraint problem, we have to convexify the last formulation with respect to the decision variable $\mathbf{P}_Q$. To this end, we can use the following identity:

$$\mathrm{Tr}[\mathbf{P}_Q\mathbf{S}_Q^T\mathbf{P}_Q^H] = \mathrm{vec}^T(\mathbf{S}_Q\mathbf{P}_Q^T)\,\mathrm{vec}(\mathbf{P}_Q^H) = \mathrm{Tr}[\mathbf{X}(\mathbf{I}\otimes\mathbf{S}_Q^T)]$$

[1] We set the subscript Q to $\mathbf{S}_Q$ to highlight its temporal nature and the fact that its size is $B \times B$. The matrices with subscript T in this paper share the common characteristic that they are $n_T \times n_T$, while those with subscript R are $n_R \times n_R$.

based on simple properties of the vectorization operator. Here, $\mathbf{X} = \mathrm{vec}(\mathbf{P}_Q^H)\,\mathrm{vec}^T(\mathbf{P}_Q^T) = \mathrm{vec}(\mathbf{P}_Q^H)\,\mathrm{vec}^H(\mathbf{P}_Q^H) \in \mathbb{C}^{n_T B\times n_T B}$. Furthermore, setting $\mathbf{Z} = \mathbf{P}_Q\mathbf{P}_Q^H$ we have

$$\begin{aligned}\mathbf{Z}_{i,k} = (\mathbf{P}_Q\mathbf{P}_Q^H)_{i,k} &= \sum_{m=1}^{B} (\mathbf{P}_Q)_{i,m}(\mathbf{P}_Q^H)_{m,k} \\ &= \sum_{m=1}^{B} \mathbf{X}_{m+(k-1)B,m+(i-1)B},\end{aligned}$$

which can be alternatively expressed as

$$\mathbf{Z} = (\mathbf{I}_{n_T}\otimes\mathbf{1}_{1\times B})\left\{(\mathbf{1}_{n_T\times n_T}\otimes\mathbf{I}_B)\odot\mathbf{X}^T\right\}(\mathbf{I}_{n_T}\otimes\mathbf{1}_{B\times 1}).$$

Here, $\odot$ denotes the Hadamard or elementwise matrix product [Horn and Johnson, 1985]. Combining the previous results, we can write (7) as:

$$\begin{aligned}\min_{\beta,\mathbf{X},\mathbf{M},\mathbf{Z}}\ & \beta \\ \text{s.t.}\ & \mathrm{Tr}[\mathbf{M}] \le \frac{\varepsilon}{\gamma},\ \begin{bmatrix} \mathbf{M} & \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{1/2} \\ \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{H/2} & \mathbf{Z}\otimes\mathbf{S}_R^{-1}\end{bmatrix} \ge 0, \\ & \mathbf{Z} = (\mathbf{I}_{n_T}\otimes\mathbf{1}_{1\times B})\left\{(\mathbf{1}_{n_T\times n_T}\otimes\mathbf{I}_B)\odot\mathbf{X}^T\right\}\cdot \\ & (\mathbf{I}_{n_T}\otimes\mathbf{1}_{B\times 1}), \\ & \mathrm{Tr}\left[\mathbf{X}\left(\mathbf{I}\otimes\mathbf{S}_Q^T\right)\right] \le \beta,\ \mathbf{X} \ge \mathbf{0},\ \mathrm{rank}\left[\mathbf{X}\right] = 1.\end{aligned}$$

This problem is nonconvex and probably NP-hard. The usual way to tackle it in the SDP literature is to drop the rank constraint. The resulting problem is a semidefinite program and can be efficiently solved by standard convex optimization packages. Upon obtaining the optimal $\mathbf{X}$, say $\mathbf{X}_*$, the rank-one solution is selected to be equal to $\sqrt{\lambda_1}\mathbf{q}_1$, where λ_1 is the greatest eigenvalue of $\mathbf{X}_*$ and $\mathbf{q}_1$ the corresponding eigenvector. We underline here that this is an intuitive but otherwise *ad hoc* solution, which has been observed to deliver good performance in practice, in the context of many such rank-one constrained problems [Manchester, 2010]. In our case, it seems that this formulation does not always yield a rank-one solution of good performance, mainly because $\sqrt{\lambda_1}\mathbf{q}_1$ has to be de-vectorized in a matrix of size $B \times n_T$, which will be equal to $\mathbf{P}_Q^H$, from which the optimal training matrix $\mathbf{P}_*$ can be easily extracted. This de-vectorization may significantly disturb the geometrical characteristics of the training sequences that will be finally transmitted by each antenna at the transmitter side.

To avoid the de-vectorization, we may instead use the following result:

$$\begin{aligned}\mathrm{Tr}\left[\mathbf{P}_Q\mathbf{S}_Q^T\mathbf{P}_Q^H\right] &= \mathrm{vec}^T\left(\mathbf{S}_Q\right)\mathrm{vec}\left(\mathbf{P}_Q^H\mathbf{P}_Q\right) \\ &= \mathrm{vec}^H\left(\mathbf{S}_Q^*\right)\mathrm{vec}\left(\mathbf{P}_Q^H\mathbf{P}_Q\right) \\ &= \left|\mathrm{vec}^H\left(\mathbf{S}_Q^*\right)\mathrm{vec}\left(\mathbf{P}_Q^H\mathbf{P}_Q\right)\right| \\ &\le \left\|\mathrm{vec}\left(\mathbf{S}_Q^*\right)\right\|\left\|\mathrm{vec}\left(\mathbf{P}_Q^H\mathbf{P}_Q\right)\right\| \\ &= \left\|\mathrm{vec}\left(\mathbf{S}_Q^*\right)\right\|\left\|\mathrm{vec}\left(\mathbf{Z}\right)\right\|,\end{aligned} \tag{8}$$

where the inequality follows from the Cauchy-Schwarz inequality and the last equality from the fact that $\|\mathrm{vec}(\mathbf{P}_Q^H\mathbf{P}_Q)\| = \|\mathrm{vec}(\mathbf{Z})\| = \|\mathbf{Z}\|_F$. Here, $\|\cdot\|_F$ denotes the Frobenius norm.

Using (8), the optimization problem (7) can be written as

$$\min_{\beta,\mathbf{Z},\mathbf{M}} \ \beta$$
$$\text{s.t.} \quad \text{Tr}[\mathbf{M}] \leq \frac{\varepsilon}{\gamma}, \quad \begin{bmatrix} \mathbf{M} & \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{1/2} \\ \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{H/2} & \mathbf{Z} \otimes \mathbf{S}_R^{-1} \end{bmatrix} \geq 0,$$
$$\|\text{vec}(\mathbf{Z})\| \leq \beta / \|\text{vec}(\mathbf{S}_Q^*)\|, \quad \mathbf{Z} = \mathbf{Z}^H \geq \mathbf{0},$$

which is a convex problem. Note that the last problem can be explicitly expressed as a semidefinite program, since the last inequality constraint on $\|\text{vec}(\mathbf{Z})\|$ can be put in the form of an LMI using the Schur complement, as follows:

$$\|\text{vec}(\mathbf{Z})\| \leq \beta / \|\text{vec}(\mathbf{S}_Q^*)\| \ \Leftrightarrow \ \begin{bmatrix} t & \text{vec}^H(\mathbf{Z}) \\ \text{vec}(\mathbf{Z}) & t\mathbf{I} \end{bmatrix} \geq 0,$$

where $t = \beta / \|\text{vec}(\mathbf{S}_Q^*)\|$.

Upon obtaining the optimal $\mathbf{Z}$, say $\mathbf{Z}_*$, the selection of the training matrix $\mathbf{P}$ has to be performed. To this end, assume that the eigenvalue decomposition (EVD) of $\mathbf{Z}_*$ is $\mathbf{U}_{Z_*}\mathbf{D}_{Z_*}\mathbf{U}_{Z_*}^H$, where $\mathbf{U}_{Z_*} \in \mathbb{C}^{n_T \times n_T}$ is its modal matrix and $\mathbf{D}_{Z_*} \in \mathbb{R}^{n_T \times n_T}$ a diagonal matrix containing its eigenvalues in decreasing order. Assume also that the EVD of $\mathbf{S}_Q^T$ is $\mathbf{U}_Q\mathbf{D}_Q\mathbf{U}_Q^H$, where $\mathbf{U}_Q \in \mathbb{C}^{B \times B}$ is its modal matrix and $\mathbf{D}_Q \in \mathbb{R}^{B \times B}$ is a diagonal matrix containing its eigenvalues in order that will be determined in the following. We denote as $\mathbf{U}_{P^*}\mathbf{D}_{P^*}\mathbf{V}_{P^*}^H$ the singular value decomposition (SVD) of $\mathbf{P}^*$. Since $\mathbf{Z} = \mathbf{P}_Q\mathbf{P}_Q^H = \mathbf{P}^*\mathbf{S}_Q^{-T}\mathbf{P}^H$, it is clear that there is an infinite number of $\mathbf{P}_Q$'s which can produce $\mathbf{Z}_*$, since if $\mathbf{P}_{Q_*}$ is such a choice then $\mathbf{P}_{Q_*}\mathbf{W}$ is also a valid choice, where $\mathbf{W} \in \mathbb{C}^{B \times B}$ is an arbitrary unitary matrix. This argument shows that our main concern with respect to the selection of $\mathbf{P}$ is the formation of $\mathbf{Z}_*$. An immediate and intuitive way to make such a choice is to select $\mathbf{U}_{P^*} = \mathbf{U}_{Z_*}$ and $\mathbf{V}_{P^*} = \mathbf{U}_Q$. This implies that $\mathbf{D}_{P^*}$ must satisfy the following relationship:

$$\mathbf{D}_{Z_*} = \mathbf{D}_{P^*}\mathbf{D}_Q^{-1}\mathbf{D}_{P^*}$$

i.e., $(\mathbf{D}_{P^*}(i,i))^2 = \mathbf{D}_{Z_*}(i,i)\mathbf{D}_Q(i,i), i = 1,\ldots,n_T$. Additionally, the ordering of the eigenvalues $\mathbf{D}_Q(i,i), i = 1,\ldots,n_T$, should be such that

$$\text{Tr}[\mathbf{P}^*\mathbf{P}^T] = \sum_{i=1}^{n_T} (\mathbf{D}_{P^*}(i,i))^2$$

is minimized. By our assumptions and using Lemma 1 in [Katselis et al., 2007], the diagonal entries of $\mathbf{D}_Q$ should be arranged in ascending order. The above choices determine the optimal $\mathbf{P}^*$, i.e., the optimal $\mathbf{P}$, say $\mathbf{P}_*$.

We may now turn our interest to the case of the MMSE channel estimator. In this case, it is immediate to see that the corresponding semidefinite program will be as follows:

$$\min_{\beta,\mathbf{Z},\mathbf{M}} \ \beta$$
$$\text{s.t.} \quad \text{Tr}[\mathbf{M}] \leq \frac{\varepsilon}{\gamma}, \quad \begin{bmatrix} \mathbf{M} & \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{1/2} \\ \boldsymbol{\mathcal{I}}_{adm}(\widehat{\mathbf{h}}_o)^{H/2} & \mathbf{R}^{-1} + \mathbf{Z} \otimes \mathbf{S}_R^{-1} \end{bmatrix} \geq 0$$
$$\|\text{vec}(\mathbf{Z})\| \leq \beta / \|\text{vec}(\mathbf{S}_Q^*)\|, \quad \mathbf{Z} = \mathbf{Z}^H \geq \mathbf{0},$$

and the choice of the optimal $\mathbf{P}$ will be exactly the same as in the case of the MVU estimator.

Remarks:

(1) The proposed choice of $\mathbf{P}$ in both cases is *one* possible solution among infinite others that achieve $\mathbf{Z}_*$. Its optimality is with respect to this aspect. However, in strict mathematical sense this choice is *ad hoc* to the same degree as the usual best rank-one approximation of $\mathbf{X}_*$ in the SDP relaxation framework. This is due to the numerical nature of the presented approach. Nevertheless, the proposed $\mathbf{P}$ is intuitive in the sense that it resembles many analytically derived optimal training matrices in the MIMO channel context, e.g., [Kotecha and Sayeed, 2004, Wong and Park, 2004, Biguesh and Gershman, 2006, Liu et al., 2007, Katselis et al., 2007, Björnson and Ottersten, 2010].

(2) In the case of the MMSE estimator, the information of $\mathbf{R}$ is indirectly encoded in the proposed training matrix $\mathbf{P}$ through the eigenvectors and the eigenvalues of the corresponding $\mathbf{Z}_*$.

(3) A thorough theoretical analysis on the distance of the obtained optimal value via the SDP relaxation from the true optimal value, i.e., a quantification of error is difficult due to the nature of the chance constraint. Such an analysis is a future theoretical direction to be pursued. Nevertheless, similar SDP relaxations to various optimization problems have been shown to deliver solutions approximately $2/\pi$ to 0.87 away from the optimal one [Manchester, 2010].

5. APPLICATIONS: MMSE CHANNEL EQUALIZATION AND ZF PRECODING

In this section we will first consider the problem of estimating a transmitted signal sequence $\{\mathbf{x}(t)\}$ from the corresponding received signal sequence $\{\mathbf{y}(t)\}$. Among a wide range of methods that are available [Paulraj et al., 2003, Verdú, 1998], we will consider the MMSE equalizer and for mathematical tractability we will approximate it by the non-causal Wiener filter. Note that for reasonably long block lengths, the MMSE estimate becomes similar to the non-causal Wiener filter [Haykin, 2001]. Thus, the optimal training design based on the non-causal Wiener filter should also provide good performance, when using an MMSE equalizer.

Let us, first, assume that $\mathbf{H}$ is available. In this ideal case, and with the transmitted signal being weakly stationary with spectrum $\mathbf{\Phi}_x$, the optimal estimate of the transmitted signal $\mathbf{x}(t)$ from the received observations of $\mathbf{y}(t)$ can be obtained according to

$$\hat{\mathbf{x}}(t;\mathbf{H}) = \mathbf{F}(q;\mathbf{H})\mathbf{y}(t)$$

where q is the unit time shift operator, i.e., $q\mathbf{x}(t) = \mathbf{x}(t{+}1)$, and the non-causal Wiener filter $\mathbf{F}(e^{j\omega};\mathbf{H})$ is given by

$$\begin{aligned}\mathbf{F}(e^{j\omega};\mathbf{H}) &= \mathbf{\Phi}_{xy}(\omega)\mathbf{\Phi}_y^{-1}(\omega) \\ &= \mathbf{\Phi}_x(\omega)\mathbf{H}^H[\mathbf{H}\mathbf{\Phi}_x(\omega)\mathbf{H}^H + \mathbf{\Phi}_n(\omega)]^{-1}.\end{aligned}$$

Here, $\mathbf{\Phi}_{xy}(\omega) = \mathbf{\Phi}_x(\omega)\mathbf{H}^H$ denotes the cross-spectrum between $\mathbf{x}(t)$ and $\mathbf{y}(t)$, and

$$\mathbf{\Phi}_y(\omega) = \mathbf{H}\mathbf{\Phi}_x(\omega)\mathbf{H}^H + \mathbf{\Phi}_n(\omega)$$

is the spectral density of $\mathbf{y}(t)$. Using our assumption that $\mathbf{\Phi}_x(\omega) = \lambda_x\mathbf{I}$, we obtain

$$\mathbf{F}(e^{j\omega};\mathbf{H}) = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \mathbf{\Phi}_n(\omega)/\lambda_x)^{-1}.$$

A simple measure of the loss of performance is the total MSE of the difference $\hat{\mathbf{x}}(t;\mathbf{H}+\widetilde{\mathbf{H}}) - \hat{\mathbf{x}}(t;\mathbf{H}) = \mathbf{\Delta}(q;\widetilde{\mathbf{H}},\mathbf{H})\mathbf{y}(t)$, where $\mathbf{\Delta}(q;\widetilde{\mathbf{H}},\mathbf{H}) \triangleq \mathbf{F}(q;\mathbf{H}+\widetilde{\mathbf{H}}) - \mathbf{F}(q;\mathbf{H})$. In view of this, we will use the channel equalization (CE) performance measure

$$J_{CE}(\widetilde{\mathbf{H}},\mathbf{H}) \triangleq \text{E}\{[\mathbf{\Delta}(q;\widetilde{\mathbf{H}},\mathbf{H})\mathbf{y}(t)]^H[\mathbf{\Delta}(q;\widetilde{\mathbf{H}},\mathbf{H})\mathbf{y}(t)]\}.$$

Note that $J_{CE}(\widetilde{\mathbf{H}},\mathbf{H})$ may be viewed as the excess MSE due to the estimation errors, of the equalized signal, compared to the case of a perfectly known channel.

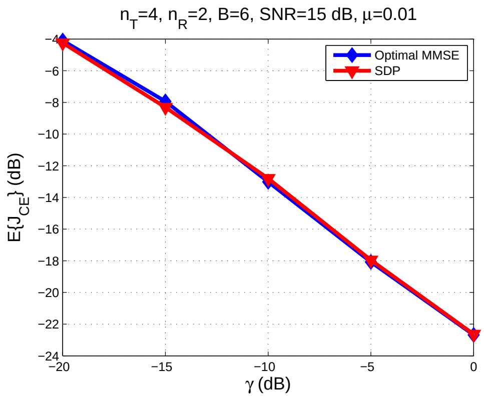

Fig. 1. $n_T = 4, n_R = 2, B = 6, \mu = 0.01$ and data transmission SNR = 15 dB: MMSE Channel Equalization based on the MMSE estimator.

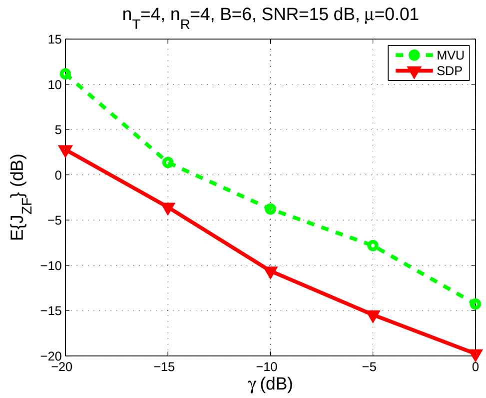

Fig. 2. $n_T = 4, n_R = 4, B = 6, \mu = 0.01$ and data transmission SNR = 15 dB: ZF precoding based on the MVU estimator.

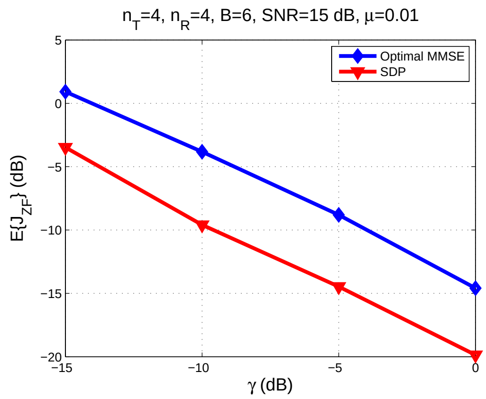

Fig. 3. $n_T = 4, n_R = 4, B = 6, \mu = 0.01$ and data transmission SNR = 15 dB: ZF precoding based on the MMSE estimator.

Apart from the receiver side channel equalization, another example of how to apply the proposed application-oriented design is the point-to-point ZF precoding, also known as channel inversion [Hochwald et al., 2005], where the channel estimate is fed back to the transmitter and its (pseudo-)inverse is used as a linear precoder. The data transmission is described by:

$$\mathbf{y}(t) = \mathbf{HBx}(t) + \mathbf{v}(t)$$

where the precoder $\mathbf{B} = \widehat{\mathbf{H}}^{\dagger}$, i.e., $\mathbf{B} = \widehat{\mathbf{H}}^{H}(\widehat{\mathbf{H}}\widehat{\mathbf{H}}^{H})^{-1}$ if we limit ourselves to the practically relevant case $n_T = n_R$ and assume that $\widehat{\mathbf{H}}$ is full rank.

Under these assumptions, we define

$$\begin{aligned}\mathbf{y}(t;\widehat{\mathbf{H}}) - \mathbf{y}(t;\mathbf{H}) &= \mathbf{H}\widehat{\mathbf{H}}^{\dagger}\mathbf{x}(t) + \mathbf{v} - (\mathbf{H}\mathbf{H}^{\dagger}\mathbf{x}(t) + \mathbf{v}) \\ &= (\widehat{\mathbf{H}}\widehat{\mathbf{H}}^{\dagger} - \widetilde{\mathbf{H}}\widehat{\mathbf{H}}^{\dagger} - \mathbf{I})\mathbf{x}(t) \simeq -\widetilde{\mathbf{H}}\mathbf{H}^{\dagger}\mathbf{x}(t).\end{aligned}$$

The cost function in this case is

$$J_{\mathrm{ZF}}(\widehat{\mathbf{H}},\mathbf{H}) \triangleq \mathrm{E}\{[\mathbf{y}(t;\widehat{\mathbf{H}}) - \mathbf{y}(t;\mathbf{H})]^H[\mathbf{y}(t;\widehat{\mathbf{H}}) - \mathbf{y}(t;\mathbf{H})]\}.$$

Lemma 1. Assuming high signal-to-noise ratio (SNR) during data transmission, $\boldsymbol{\mathcal{I}}_{adm} = \lambda_x \mathbf{I} \otimes \left(\mathbf{H}\mathbf{H}^H\right)^{-1}$ for $n_T \geq n_R$ in the MMSE channel equalization case and $\boldsymbol{\mathcal{I}}_{adm} = \lambda_x \mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H\right)^{-2} \mathbf{H} \otimes \mathbf{I}$ for $n_T = n_R$ in the ZF precoding case.

The proof of this result is long, hence it is omitted due to reasons of space.

6. NUMERICAL EXAMPLES

The purpose of this section is to examine the performance of optimal training sequence designs, and to compare them with other methods. In all figures, fair comparison among the presented schemes is ensured via training energy equalization. Additionally, the matrices $\mathbf{R}_T, \mathbf{R}_R, \mathbf{S}_Q, \mathbf{S}_R$ follow the exponential model, that is, they are built according to

$$(\mathbf{R})_{i,j} = r^{j-i}, \;\; j \geq i,$$

where r is the (complex) normalized correlation coefficient with magnitude $\rho \triangleq |r| < 1$. We choose to examine the most intriguing case of $|r|$ for all the presented schemes. Therefore, in all plots $|r| = 0.9$ for all matrices $\mathbf{R}_T, \mathbf{R}_R, \mathbf{S}_Q, \mathbf{S}_R$. Additionally, the transmit SNR during data transmission is chosen to be 15 dB. High SNR expressions given by Lemma 1 are therefore used for optimal training sequence designs. We assume that the channel changes from block to block according to the relationship $\mathbf{H}_i = \mathbf{H}_{i-1} + \mu\mathbf{E}_i$, where $\mathbf{E}_i$ is built as $\mathbf{H}$ and it is completely independent from $\mathbf{H}_{i-1}$. J_{CE} is averaged over multiple realizations of $\mathbf{H}_i$ with depth equal to 5. For initialization, the "SDP" scheme in Figs. 1 and 3 uses the Optimal MMSE estimate derived in [Björnson and Ottersten, 2010] and the "SDP" scheme in Fig. 2 the optimal Gauss-Markov estimate presented in [Katselis et al., 2008].

In Fig. 1, $E\{J_{CE}\}$ versus the accuracy γ is presented for two different schemes. The "Optimal MMSE" scheme is presented in [Björnson and Ottersten, 2010], and corresponds to the optimal training for the MMSE estimator with respect to the channel estimation error. This scheme corresponds to the state of the art in communication systems at the moment. The "SDP" scheme is the one presented in this paper for the MMSE estimator. For the J_{CE} performance metric, the application-oriented design demonstrates approximately the same performance with the the "Optimal MMSE" scheme. This is due to the fact that $\boldsymbol{\mathcal{I}}_T = \mathbf{I}$ for this particular application. The application-oriented design demonstrates considerably better performance for all γ-values in the case of the ZF precoding application, as we can see in Fig. 3. In this case, $\boldsymbol{\mathcal{I}}_T \neq \mathbf{I}$.

As far as Fig. 2 is concerned, the scheme "MVU" is the MVU estimator with optimal training for channel estimation purposes. This scheme is usually used in communication systems and has been presented in [Katselis et al., 2008]. The scheme "SDP" is the SDP relaxation solution for the MVU estimator presented in this paper. We observe that the application-oriented design demonstrates again better performance for all γ-values in the case of the ZF precoding application.

7. CONCLUSIONS

Semidefinite programming relaxations for the chance constraint input design problem based on the Markov bound have been presented in this paper. The derived SDP formulations were used in the context of MMSE channel equalization and ZF precoding in MIMO communication systems. Numerical results have verified that the application-oriented input design provides a powerful experiment design framework, when combined with SDP relaxation tools.

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